

HW SOL 4.4

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Name: _____

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Math 9 Enriched: Section 4.4 Factoring & Solving Trinomials

1. Factor each of the following expressions:

a. $2x^2 + 5x + 2$
 $\begin{array}{r} 2 \quad -2 = 4 \\ 1 \quad -1 = 1 \end{array}$
 $(2x+1)(x+2)$

b. $4x^2 + 9x + 2$
 $\begin{array}{r} 4 \quad -2 = 8 \\ 1 \quad -1 = 1 \end{array}$
 $(4x+1)(x+2)$

c. $21x^2 + 17x - 30$
 $\begin{array}{r} 3 \quad 5 = 15 \\ 7 \quad 6 = 42 \end{array}$
 $(3x+5)(7x+6)$

d. $2x^2 - 11x + 15$
 $\begin{array}{r} 2 \quad -3 = 6 \\ 1 \quad -5 = -5 \end{array}$
 $(2x-5)(x-3)$

e. $8x^4 - 14x^2y^2 + 3y^4$
 $\begin{array}{r} 4 \quad -3 = -12 \\ 2 \quad -1 = -2 \end{array}$
 $(4x^2 - y^2)(2x^2 - 3y^2)$

f. $15x^2 - 28x - 32$
 $\begin{array}{r} 3 \quad 16 = 48 \\ 5 \quad 2 = 10 \end{array}$
 $(3x-8)(5x+4)$

g. $2m^3n - m^2n - 21mn$
 $mn(2m^2 - m - 21)$
 $\begin{array}{r} 2 \quad 3 = 6 \\ 1 \quad -7 = -7 \end{array}$
 $mn(2m-7)(m+3)$

h. $10x^2 - 33xy - 7y^2$
 $\begin{array}{r} 2 \quad -7 = -14 \\ 5 \quad 1 = 5 \end{array}$
 $(2x+y)(5x-7y)$

i) $16mn - 4m^2 + 28n - 7m$
 $4m(4n-m) + 7(4n-m)$
 $(4m+7)(4n-m)$

j) $4xy + 6 - x - 24y$
 $\begin{array}{r} 4xy - x + 6 - 24y \\ x(4y-1) + 6(1-4y) \\ x(4y-1) - 6(4y-1) \\ (4y-1)(x-6) \end{array}$

k) $21xy - 12b^2 + 14xb - 18by$
 $21xy + 14xb - 12b^2 - 18by$
 $7x(3y+2b) - 6b(2b+3y)$
 $(7x-6b)(3y+2b)$
 [NOTE: $3y+2b = (2b+3y)$]

l) $28xy + 25 + 35x + 20y$
 $28xy + 35x + 20y + 25$
 $7x(4y+5) + 5(4y+5)$
 $(7x+5)(4y+5)$

2. Factor and solve each of the following expressions:

a. $x^2 - 13x + 40 = 0$
 $(x-5)(x-8) = 0$
 $x-5=0 \quad x-8=0$
 $x=5 \quad x=8$

b. $x^2 - x + 56 = -13$
 $a=1 \quad b=-1 \quad c=69$
 $x^2 - x + 69 = 0$
 NOT FACTOR

c. $x^3 + 4x^2 - 12x = 0$
 $x(x^2 + 4x - 12) = 0$
 $x(x+6)(x-2) = 0$
 $x=0 \quad x=-6 \quad x=2$

d. $x(x+2) = 2x(x-3)$
 $x(x+2) - 2x(x-3) = 0$
 $x[(x+2) - 2(x-3)] = 0$
 $x[x+2-2x+6] = 0$
 $x[8-x] = 0 \rightarrow x=0, x=8$

e. $x = x^2(x+5)$
 $0 = x^2(x+5) - x$
 $0 = x[x(x+5) - 1]$
 $0 = x[x^2 + 5x - 1]$
 $a^2 + b^2 + c = 0$
 $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

f. $(x+2) = (x+2)(x-4)$

g) $x^3 + 4x^2 - 12x = 0$

h) $2y^3 + 6y^2 - 108y = 0$

i) $12y^3 - 21y^2 + 28y - 49 = 0$
 $3y^2(4y-7) + 7(4y-7) = 0$
 $(4y-7)(3y^2 + 7) = 0$
 $\downarrow \quad \downarrow$
 $y = \frac{7}{4} \quad \text{No soln}$

j) $105y^3 + 175y^2 - 75y = 125$

k) $96n^4 - 84n^3 + 112n^2 = 98n$

l) $24x^4 + 15x^3 = 56x^2 + 35x$

3. For what value(s) of "k" can each trinomial be factored?

a. $4x^2 + kx + 3$

Handwritten work for (a):
 $2 \times -1 = -2$
 $2 \times -3 = -6$
 $2 \times 8 = 16$
 $2 \times -8 = -16$
 $k = \pm 8$

b. $4x^2 + kx + 25$

Handwritten work for (b):
 $4 \times -1 = -4$
 $1 \times -3 = -3$
 $k = \pm 7$
 $4 \times 1 = 4$
 $1 \times 3 = 3$
 $k = \pm 13$

c. $6x^2 + kx - 9$

4. What value of "x" satisfies $x(x - 2009) = x(x + 2009)$?

5. What are all values of "x" for which $x\sqrt{2} = 2\sqrt{x}$?

6. What is the only pair of real numbers (a,b) for which the equation is equal $a^3 + ab^2 = 30$ and $b^3 + ba^2 = 90$?

Handwritten solution for problem 6:

① $a(a^2 + b^2) = 30$ ② $b(b^2 + a^2) = 90$

$\frac{b(b^2 + a^2)}{a(b^2 + a^2)} = \frac{90}{30}$

$\frac{b}{a} = 3$

$b = 3a$

$a(a^2 + 9a^2) = 30$

$a(10a^2) = 30$

$a^3 = 3$

$a = \sqrt[3]{3}$

$b = 3(\sqrt[3]{3})$

7. Factor the following completely: $x^5 + x^4 + x^3 + x^2 + x + 1$

$$x^4 + 2x^3 + 1 = (x^2+1)(x^2+1)$$

$$\begin{aligned} x^4(x+1) + x^2(x+1) + 1(x+1) &= (x+1) \left[(x^2+1)^2 - x^2 \right] \\ (x+1) \left[\underline{x^4 + x^2 + 1} \right] &= (x+1) \left[(x^2+1-x)(x^2+1+x) \right] \\ (x+1)(x^4 + \underline{x^2+1} + \underline{x^2-x^2}) & \\ (x+1)(x^4 + 2x^2 + 1 - x^2) & \end{aligned}$$

8. Write $4x^2 - 9y^2 + 4x^3 + 6x^2y$ as a product of two non-constant polynomials with integral coefficients.

9. Two different circles that pass through the point (1,3) are tangent to both coordinate axes. If the length of the radius of the smaller circle is "r" and the length of the radius of the larger circle is "R", what is the value of "r + R"?

① Circle EQN:

$$x^2 + y^2 = R^2 \rightarrow (x-h)^2 + (y-k)^2 = R^2$$

(h, k) CENTER OF THE CIRCLE.

$$\begin{aligned} (x-R)^2 + (y-R)^2 = R^2 &\rightarrow (1-R)^2 + (3-R)^2 = R^2 \\ (x-r)^2 + (y-r)^2 = r^2 &\rightarrow (1-r)^2 + (3-r)^2 = r^2 \\ (1-R)^2 - (1-r)^2 + (3-R)^2 - (3-r)^2 &= R^2 - r^2 \\ (1-R+1-r)(1-R-1+r) + (3-R+3-r)(3-R-3+r) &= (R+r)(R-r) \\ (2-R-r)(-R+r) + (6-R-r)(-R+r) &= (R+r)(R-r) \\ -2(R+r) - 6(R+r) &= (R+r) \\ R+r &= 8 \end{aligned}$$

10. For what integer "n" are the roots of $x^2 - 7x + n = 0$ consecutive integers?

11. If $(x-10)(x+10) = 0$, what is the value of $(x-1)(x+1)$?

12. What is the only value of "x" which satisfies $(x - 2005)^2 = (x + 2005)^2$?

13. Which positive integer "n" satisfies $n^{2006} + 2n^{2007} = 3$

$$n^{2006}(1 + 2n) = 3$$

$$1^{2006}(1 + 2(1)) = 3$$

$$1 \times 3 = 3$$

14. If r_1, r_2, r_3, r_4 are the roots of $x^4 - 4x^2 + 2 = 0$, what is the value of $(1+r_1)(1+r_2)(1+r_3)(1+r_4)$?

$$a=1 \quad b=-4 \quad c=2$$

$$x^2 = \frac{4 \pm \sqrt{16 - 4(1)(2)}}{2}$$

$$x^2 = \frac{4 \pm \sqrt{8}}{2}$$

$$x^2 = \frac{4 \pm 2\sqrt{2}}{2}$$

$$x^2 = 2 \pm 2\sqrt{2}$$

$$x^2 = 2 + 2\sqrt{2} \quad \text{or} \quad x^2 = 2 - 2\sqrt{2}$$

$$r_1 x = +\sqrt{2+2\sqrt{2}} \quad r_2 x = +\sqrt{2-2\sqrt{2}}$$

$$r_3 x = -\sqrt{2+2\sqrt{2}} \quad r_4 x = -\sqrt{2-2\sqrt{2}}$$

$$(a+b)(a-b)$$

$$a^2 - b^2$$

15. Challenge: Determine all values of "x" for which $(x^2 + 3x + 2)(x^2 - 2x - 1)(x^2 - 7x + 12) + 24 = 0$

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$$(x+1)(x+2)[x^2-2x-1](x-3)(x-4) + 24 = 0$$

$$(x+1)(x-3)[x^2-2x-1](x+2)(x-4) + 24 = 0$$

$$[x^2-2x-3][x^2-2x-1](x^2-2x-8) + 24 = 0$$

$$(A-3)(A-1)(A-8) + 24 = 0$$